

***CPT* INVARIANCE TESTS IN NEUTRAL KAON DECAY**

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CPT theorem is based on three assumptions: quantum field theory, locality, and Lorentz invariance, and thus it is a fundamental probe of our basic understanding of particle physics. Strangeness oscillation in $K^0 - \bar{K}^0$ system, described by the equation

$$i \frac{d}{dt} \begin{bmatrix} K^0 \\ \bar{K}^0 \end{bmatrix} = [M - i\Gamma/2] \begin{bmatrix} K^0 \\ \bar{K}^0 \end{bmatrix} ,$$

where M and Γ are hermitian matrices (see PDG review [1], references [2,3], and KLOE paper [5] for notations and previous literature), allows a very accurate test of *CPT* symmetry; indeed since *CPT* requires $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$, the mass and width eigenstates, $K_{S,L}$, have a *CPT*-violating piece, δ , in addition to the usual *CPT*-conserving parameter ϵ :

$$\begin{aligned} K_{S,L} &= \frac{1}{\sqrt{2(1+|\epsilon_{S,L}|^2)}} \left[(1 + \epsilon_{S,L}) K^0 \pm (1 - \epsilon_{S,L}) \bar{K}^0 \right] \\ \epsilon_{S,L} &= \frac{-i\Im(M_{12}) - \frac{1}{2}\Im(\Gamma_{12}) \mp \frac{1}{2} \left[M_{11} - M_{22} - \frac{i}{2}(\Gamma_{11} - \Gamma_{22}) \right]}{m_L - m_S + i(\Gamma_S - \Gamma_L)/2} \\ &\equiv \epsilon \pm \delta. \end{aligned} \tag{1}$$

Using the phase convention $\Im(\Gamma_{12}) = 0$, we determine the phase of ϵ to be $\varphi_{SW} \equiv \arctan \frac{2(m_L - m_S)}{\Gamma_S - \Gamma_L}$. Imposing unitarity to an arbitrary combination of K^0 and \bar{K}^0 wave functions, we obtain the Bell-Steinberger relation [4] connecting *CP* and *CPT* violation in the mass matrix to *CP* and *CPT* violation in the decay; in fact, neglecting $\mathcal{O}(\epsilon)$ corrections to the coefficient of the *CPT*-violating parameter, δ , we can write [5]

$$\begin{aligned} \frac{\Gamma_S + \Gamma_L}{\Gamma_S - \Gamma_L} + i \tan \phi_{SW} \left[\frac{\Re(\epsilon)}{1 + |\epsilon|^2} - i\Im(\delta) \right] = \\ \frac{1}{\Gamma_S - \Gamma_L} \sum_f A_L(f) A_S^*(f), \end{aligned} \tag{2}$$

where $A_{L,S}(f) \equiv A(K_{L,S} \rightarrow f)$. We stress that this relation is phase-convention-independent. The advantage of the neutral kaon system is that only a few decay modes give significant contributions to the r.h.s. in Eq. (2); in fact, defining for the hadronic modes

$$\alpha_i \equiv \frac{1}{\Gamma_S} \langle \mathcal{A}_L(i) \mathcal{A}_S^*(i) \rangle = \eta_i \mathcal{B}(K_S \rightarrow i),$$

$$i = \pi^0 \pi^0, \pi^+ \pi^-(\gamma), 3\pi^0, \pi^0 \pi^+ \pi^-(\gamma), \quad (3)$$

the recent data from CPLEAR, KLOE, KTeV, and NA48 have led to the following determinations (the analysis described in Ref. 5 has been updated by using the recent measurements of K_L branching ratios from KTeV [6,7], NA48 [8,9], and the results described in the CP violation in K_L decays minireview, and the recent KLOE result [10])

$$\alpha_{\pi^+ \pi^-} = ((1.112 \pm 0.010) + i(1.061 \pm 0.010)) \times 10^{-3},$$

$$\alpha_{\pi^0 \pi^0} = ((0.493 \pm 0.005) + i(0.471 \pm 0.005)) \times 10^{-3},$$

$$\alpha_{\pi^+ \pi^- \pi^0} = ((0 \pm 2) + i(0 \pm 2)) \times 10^{-6},$$

$$|\alpha_{\pi^0 \pi^0 \pi^0}| < 1.5 \times 10^{-6} \quad \text{at 95\% CL.} \quad (4)$$

The semileptonic contribution to the right-handed side of Eq. (2) requires the determination of several observables: we define [2,3]

$$\mathcal{A}(K^0 \rightarrow \pi^- l^+ \nu) = \mathcal{A}_0(1 - y),$$

$$\mathcal{A}(K^0 \rightarrow \pi^+ l^- \nu) = \mathcal{A}_0^*(1 + y^*)(x_+ - x_-)^*,$$

$$\mathcal{A}(\bar{K}^0 \rightarrow \pi^+ l^- \nu) = \mathcal{A}_0^*(1 + y^*),$$

$$\mathcal{A}(\bar{K}^0 \rightarrow \pi^- l^+ \nu) = \mathcal{A}_0(1 - y)(x_+ + x_-), \quad (5)$$

where x_+ (x_-) describes the violation of the $\Delta S = \Delta Q$ rule in CPT -conserving (violating) decay amplitudes, and y parametrizes CPT violation for $\Delta S = \Delta Q$ transitions. Taking advantage of their tagged $K^0(\bar{K}^0)$ beams, CPLEAR has measured $\Im(x_+)$, $\Re(x_-)$, $\Im(\delta)$, and $\Re(\delta)$ [11]. These determinations have been improved in Ref. 5 by including the

information $A_S - A_L = 4[\Re(\delta) + \Re(x_-)]$, where $A_{L,S}$ are the K_L and K_S semileptonic charge asymmetries, respectively, from the PDG [12] and KLOE [13]. Here we are also including the T -violating asymmetry measurement from CPLEAR [14].

Table 1: Values, errors, and correlation coefficients for $\Re(\delta)$, $\Im(\delta)$, $\Re(x_-)$, $\Im(x_+)$, and $A_S + A_L$ obtained from a combined fit, including KLOE [5] and CPLEAR [14].

| | value | Correlations coefficients | | | |
|---------------|-----------------------------------|---------------------------|-------|------|------|
| $\Re(\delta)$ | $(3.0 \pm 2.3) \times 10^{-4}$ | 1 | | | |
| $\Im(\delta)$ | $(-0.66 \pm 0.65) \times 10^{-2}$ | -0.21 | 1 | | |
| $\Re(x_-)$ | $(-0.30 \pm 0.21) \times 10^{-2}$ | -0.21 | -0.60 | 1 | |
| $\Im(x_+)$ | $(0.02 \pm 0.22) \times 10^{-2}$ | -0.38 | -0.14 | 0.47 | 1 |
| $A_S + A_L$ | $(-0.40 \pm 0.83) \times 10^{-2}$ | -0.10 | -0.63 | 0.99 | 0.43 |

The value $A_S + A_L$ in Table 1 can be directly included in the semileptonic contributions to the Bell Steinberger relations in Eq. (2)

$$\begin{aligned}
 & \sum_{\pi\ell\nu} \langle \mathcal{A}_L(\pi\ell\nu) \mathcal{A}_S^*(\pi\ell\nu) \rangle \\
 &= 2\Gamma(K_L \rightarrow \pi\ell\nu) (\Re(\epsilon) - \Re(y) - i(\Im(x_+) + \Im(\delta))) \\
 &= 2\Gamma(K_L \rightarrow \pi\ell\nu) ((A_S + A_L)/4 - i(\Im(x_+) + \Im(\delta))) . \quad (6)
 \end{aligned}$$

Defining

$$\alpha_{\pi\ell\nu} \equiv \frac{1}{\Gamma_S} \sum_{\pi\ell\nu} \langle \mathcal{A}_L(\pi\ell\nu) \mathcal{A}_S^*(\pi\ell\nu) \rangle + 2i \frac{\tau_{K_S}}{\tau_{K_L}} \mathcal{B}(K_L \rightarrow \pi\ell\nu) \Im(\delta) , \quad (7)$$

we find:

$$\alpha_{\pi\ell\nu} = ((-0.2 \pm 0.5) + i(0.1 \pm 0.5)) \times 10^{-5} .$$

Inserting the values of the α parameters into Eq. (2), we find

$$\begin{aligned}
 \Re(\epsilon) &= (161.1 \pm 0.5) \times 10^{-5}, \\
 \Im(\delta) &= (-0.7 \pm 1.4) \times 10^{-5}. \quad (8)
 \end{aligned}$$

The complete information on Eq. (8) is given in Table 2.

Table 2: Summary of results: values, errors, and correlation coefficients for $\Re(\epsilon)$, $\Im(\delta)$, $\Re(\delta)$, and $\Re(x_-)$.

| | value | Correlations coefficients | | | |
|-----------------|----------------------------------|---------------------------|-------|-------|---|
| $\Re(\epsilon)$ | $(161.1 \pm 0.5) \times 10^{-5}$ | + 1 | | | |
| $\Im(\delta)$ | $(-0.7 \pm 1.4) \times 10^{-5}$ | + 0.09 | 1 | | |
| $\Re(\delta)$ | $(2.4 \pm 2.3) \times 10^{-4}$ | + 0.08 | -0.12 | 1 | |
| $\Re(x_-)$ | $(-4.1 \pm 1.7) \times 10^{-3}$ | + 0.14 | 0.22 | -0.43 | 1 |

Now the agreement with *CPT* conservation, $\Im(\delta) = \Re(\delta) = \Re(x_-) = 0$, is at 18% C.L.

The allowed region in the $\Re(\epsilon) - \Im(\delta)$ plane at 68% CL and 95% C.L. is shown in the top panel of Fig. 1.

The process giving the largest contribution to the size of the allowed region is $K_L \rightarrow \pi^+\pi^-$, through the uncertainty on ϕ_{+-} .

The limits on $\Im(\delta)$ and $\Re(\delta)$ can be used to constrain the $K^0 - \bar{K}^0$ mass and width difference

$$\delta = \frac{i(m_{K^0} - m_{\bar{K}^0}) + \frac{1}{2}(\Gamma_{K^0} - \Gamma_{\bar{K}^0})}{\Gamma_S - \Gamma_L} \cos \phi_{SW} e^{i\phi_{SW}} [1 + \mathcal{O}(\epsilon)].$$

The allowed region in the $\Delta M = (m_{K^0} - m_{\bar{K}^0})$, $\Delta\Gamma = (\Gamma_{K^0} - \Gamma_{\bar{K}^0})$ plane is shown in the bottom panel of Fig. 1. As a result, we improve on the previous limits (see for instance, P. Bloch in Ref. 12) and in the limit $\Gamma_{K^0} - \Gamma_{\bar{K}^0} = 0$ we obtain

$$-4.0 \times 10^{-19} \text{ GeV} < m_{K^0} - m_{\bar{K}^0} < 4.0 \times 10^{-19} \text{ GeV} \quad \text{at } 95 \% \text{ C.L.}$$

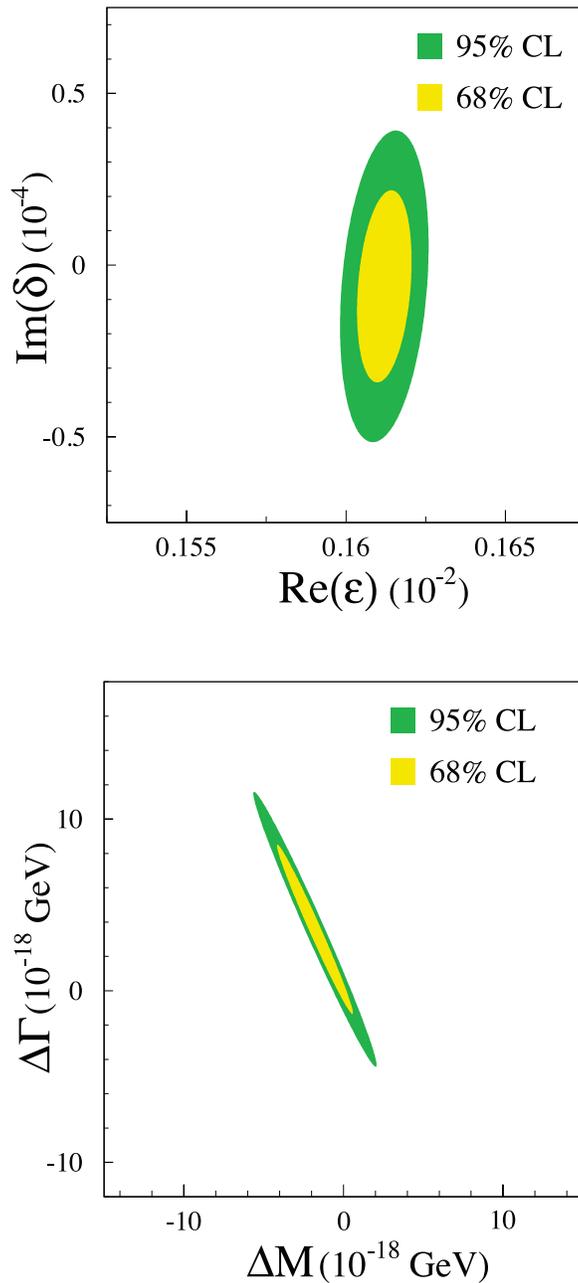


Figure 1: Top: allowed region at 68% and 95% C.L. in the $\Re(\epsilon)$, $\Im(\delta)$ plane. Bottom: allowed region at 68% and 95% C.L. in the $\Delta M, \Delta\Gamma$ plane.

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